Load-Life-Damage Hybrid Approach for Substantiation of Composite Aircraft Structures

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ABSTRACT

The primary goal in a damage-tolerance certification program is to avoid catastrophic failure due to fatigue, corrosion, or accidental damage throughout the operational life of the aircraft. The damage-tolerance philosophy is well established for metallic airframes, where proven methods (structural analysis and inspection procedures) and supporting databases exist to detect damage and predict crack growth and residual strength. However, damage characteristics, inspection procedures, analysis methods, and experimental databases are not well understood to apply the damage-tolerance philosophy to composite structures. Thus, there is a growing interest in damage-tolerance methodology to determine the fatigue life of composite structures under repeated loading. The state of damage within a composite structure is complex and dependent on a number of variables that define the intrinsic properties of the sandwich constructions and the extrinsic damage-causing event. Further, the barely visible impact damage (BVID), allowable damage limit (ADL), and critical damage threshold (CDT) are not clearly defined in terms of a rational damage metric. Traditionally, visual inspection procedures have been used for detecting damage in composite structures (in-service); hence, BVID came into existence. The current definitions of BVID are based on the residual indentation depth, which has been clearly shown to be configuration-dependent and often misleading. Another issue coupled with this is the choice of NDI techniques, which dictates the damage metric defining the BVID criterion. An extensive scatter analysis was conducted to support U.S. Navy F/A-18 certification on several material databases. The first phase of this research discussed extension of the methodologies to new material systems and construction techniques. Because these tests are usually the most expensive to performed during the certification process, the goal of the program is to provide an efficient certification approach that weighs both the economic aspects of certification and the time frame required for certification testing, while ensuring that safety is the key priority. The goal of this phase of the research was to develop a probabilistic approach to synthesizing the life factor, load-enhancement factors, and damage in composite structures in order to determine the fatigue life of a damage-tolerant aircraft. This methodology was extended to the current certification approach to explore extremely improbable high-energy impact threats, i.e., damages that reduce the residual strength of aircraft to limit-load capability and allow incorporating certain design changes into full-scale substantiation without the burden of additional timeconsuming and costly tests.

KEY WORDS: scatter, load enhancement, life factor, certification, composite, fatigue, damage

1. INTRODUCTION

Over the past 25 years, the use of advanced composite materials in aircraft primary structures has increased significantly. In 1994, with the Advanced General Aviation Transport Experiments (AGATE) program, the National Aeronautics and Space Administration (NASA) and the Federal Aviation Administration (FAA) revitalized the use of composites in general and commercial aviation. Driven by the demand for fuel-efficient, light-weight, and high-stiffness structures that have fatigue durability and corrosion resistance, modern large commercial aircraft are designed with more than 50 percent composite materials. Although there are key differences between metal and composite damage mechanics and durability concerns, the certification philosophy for composites must meet structural integrity, safety, and durability requirements. Despite the many advantages, composite structural certification becomes challenging due to the lack of experience in large-scale structures, complex interactive failure mechanisms, sensitivity to temperature and moisture, and scatter in the data, especially in fatigue. The overall objective of this research was to provide guidance into structural substantiation of composite airframe structures through an efficient approach that weighs both the economic aspects of certification and the timeframe required for testing, while ensuring safety.

Most current fatigue life-assessment methodologies for advanced composite structures rely on empirical S-N data in the lower levels of the building blocks of testing. Variation of material characteristics for different fiber-resin systems, layup configurations, environments, loading conditions, etc. often make the analysis and testing of composites challenging. Anisotropic heterogeneous characteristics and change in failure modes over the fatigue life as well as multiple failure mechanisms that interact with each other make it challenging to predict damage growth in composite structures. Consequently, most of the damage mechanisms and wearout approaches discussed in the literature also depend on empirical data for refinement or calibration. Some approaches only discuss failure progression under certain loading configurations and often specific to a material system. With increased use of composite materials in primary structures, there is a growing need to investigate extremely improbable high-energy impact threats that reduce the residual strength of a composite structure to limit load. Currently, this issue is not explicitly addressed in full-scale substantiation, and no fatigue requirements exist, i.e., only "get home" loads. The methodology outlined in this research phase can be applied to further investigate the large damage threats, while mitigating risks of unintentional structural failure during full-scale test.

2. RELIABILITY ANALYSIS

2.1 Reliability (Scatter) Analysis for Fatigue Life Determination

Over the years, composite and hybrid structural certification programs have adopted methodologies utilized for metal structures that are based on several decades of experience in full-scale structural certification and service. Despite the advantages, such as high specific weight, tailorability, and fatigue resistance, composite structural certification becomes challenging due to the lack of experience with large-scale structures, complex interactive failure mechanisms, sensitivity to temperature and moisture, and scatter in the data, especially relative to fatigue. Despite the superior fatigue properties, the heterogeneous nature due to the presence of multiple constituents and the notch sensitivity of composites resulted in special requirements

for durability and damage tolerance (DaDT) substantiation of composite structures. The data scatter in composites is related to the reliability of composite structure through a concept known as the scatter factor, which signifies the relation between the central tendency of a data set (mean) and the extreme statistics (allowables). In order to develop a certification methodology for composite structures that has the same level of reliability as observed in metal certification approaches, accounting for the inherent difference between metal and composites, the FAA and U.S. Navy developed a certification approach for bolted composite structures [1,2] as part of the F/A-18 certification. This methodology was formulated to account for the uncertainties of applied loads as well as the scatter in static strength and fatigue life related to composite structures. This approach analyzes the data scatter in the static (residual) strength and fatigue life of composites to establish a certification methodology that has the same level of reliability as for metal structures. Furthermore, this approach attempts to address the issues related to hybrid (composite and metallic) structures through a combined approach referred to as the load-life approach, which will be further discussed in this paper. This approach was developed for what, at that time, was current composite usage and did not explicitly account for the damage in composite structures or adhesively bonded structural details. Over the years, several composite structural certification programs employed this certification methodology, which was developed for materials and test methods that were considered current at the time. Since then, materials and process techniques as well as test methods for evaluating composites have evolved. Consequently, test data often display significantly less scatter with high reliability. Thus, the probabilistic approach employed by Whitehead, et al., [1] can be reevaluated for newer material forms and to represent structural details of current aircraft structures to obtain improved life and load-enhancement factors [3,4,5]. Figure 1 shows the outline of this approach and its application to a full-scale DaDT substantiation of composite structures.

3. LOAD-LIFE-DAMAGE HYBRID APPROACH

During full-scale fatigue testing, it is common to use a combination of the life factor and loadenhancement factors. This research proposes the generation of these factors for a specific certification program using design details, such as materials, layup, loading conditions, etc., that are related to the composites structure, rather than using the factors generated for U.S. Navy F/A-18 certification [1] by pooling material test data from several databases that were then current. As shown in references [4, 5], the scatter in the composite data is reduced considerably due to the improvements in materials, process technologies, and composite test methods. Furthermore, it is shown that the scatter in composite data tends to be less for notched or damaged specimens. In reference [5], the composite data scatter was evaluated based on the extent of the damage. When a DaDT article is impacted with certain damage, such as category 3 as defined in reference [6], the following assumptions can be made with a high degree of confidence:

- 1. The impact-damaged region becomes the critical location of the structure.
- 2. Imminent damage initiation at this location will cause structural collapse or load distribution that can be predicted by analysis for subsequent test validation.



Figure 1. Derivation and application of life and load-enhancement factors through scatter analysis of composite test data.

Then elements or sub-components that represent the details of the impacted location can be tested to obtain a new life shape parameter for that particular structural detail and critical load conditions associated with the failure mode. The first condition is essential, as the modal life shape parameter (MLSP) that was obtained as the procedure outlined in Figure 1 is replaced by the shape parameter of the fatigue analysis of element or sub-component tests, i.e., the analysis conducted on impact-damaged elements. The second condition is required, as the failure mode of the structure is assumed to occur as a direct result of damage instigation of impact damage. If load is redistributed instead of complete structural failure, then the use of a newly defined life shape parameter must be superseded by corresponding life shape parameters of the subsequent damage state of the structure for the remainder of the test, i.e., if category 2 damage is transferred to a category 3 damage as a result of the damage propagation, then the remainder of the DaDT test requires the use of a category 3 life shape parameter instead of a category 2 life shape parameter. DaDT element or sub-component tests must be designed to address the

expected outcome. This approach is graphically illustrated in Figure 2 for a full-scale structural test that was initiated with standard LEFs for the durability phase and then continued using load-life-damage (LLD) hybrid approach for the DaDT phase.

Based on the design analysis and strain surveys, the most critical locations of the structure are selected for damage infliction. Then, the impact parameters are determined to inflict a certain damage, i.e., category 2 or category 3. This step requires an analysis of the local details such as materials, laminate stacking sequence, critical loading modes, etc. For example, a nonlinear finite element analysis of the local details with a continuum damage model, i.e., stiffness degradation due to fiber/matrix cracking and plastic deformation under shear loading, and a contact algorithm to model surface erosion (element removal) on multiple contact bodies during impact, i.e., impactor and the contact surface as well as the interior ply interfaces, can be used to determine the extent of the damage for a particular impact scenario. A model with such details requires significant computing time and often extremely sensitive to the mesh density, element type, input required for the damage model, etc. Therefore, scaled element tests are recommended, where possible, not only to cross-examine the impact parameters prior to impacting the full-scale test article but also to validate/calibrate analytical models. The element test also can be used for the scatter analysis of a particular damage scenario. Consequently, the LEFs and life factor corresponding to the selected damage scenario can be calculated for the damage-tolerance phase of the full-scale test.



Figure 2. DaDT test demonstration with Load-Life-Damage hybrid approach [4].

As shown in Figure 2, the LLD hybrid approach focus on the most critical details of the structure and interpret the structural and loads details into the most representative repeated load testing in element level to gain information on residual strength, fatigue sensitivity, inspection methods, and inspection intervals during full-scale test substantiation. Typically, a critical damage such as a category 3 is readily detectible during a short walk-around inspection. In the safety stand point, the goal is to focus on the most critical yet least detectible damage that may occur during service. This may be a category 2 or 3 depending on the detectability and the inspection methods that can be practically applied, i.e., short walk-around inspection or a scheduled inspection at heavy maintenance.

In order to demonstrate the application of LLD hybrid approach, the element test data in references [4] were used, considering only the effects of impact damage on the fatigue life scatter, and three sets of LEFs were generated with respect to the extent of the damage and combined with the original AS4-PW LEFs to generate a surface plot of LEFs, as shown in Figure 3. First, the LEF corresponding to three DLTs using AS4-PW data was selected but the test was only conducted up to two DLTs. Then, the structure was impacted with a LID and the corresponding LEF curve was used to select the LEF for the remainder of the test. The LLD approach introduces the use of multiple LEFs for a particular composite structure, based on the damage category, i.e., use of different LEF curves representing different damage severities.

When multiple LEF (load-life) curves are used for different damage scenarios, a concept called the load-life shift was introduced in reference [4] to calculate the remaining test duration upon introduction of the new damage to test article. The load-life shift given in equation (1) calculates the remaining test duration based on the percentage of unsubstantiated design life in the previous test phase.

$$N_2^T = \left(1 - \frac{N_1^T}{N_1^R}\right) \cdot N_2^R \tag{1}$$

In equation (1), the subscripts *I* and *2* correspond to the test phase, and the superscripts *R* and *T* denote the corresponding repeated life for a particular LEF and the actual test duration, respectively, to demonstrate the reliability of design lifetime. For example, the test duration of 3 DLTs (N_1^R) from AS4-PW curve corresponds to an LEF of 1.033 (Figure 3). The test is conducted for 2 DLTs (N_1^T) , and the structure is inflicted with a LID. The test duration of 2.5 DLTs (N_2^R) from LID curve corresponds to an LEF of 1.014 (Figure 3). Since 2 out of 3 DLTs required in the first phase of the test is completed, using equation (1), the remaining test duration (for phase 2) is calculated as 0.83 DLT (N_2^T) . Therefore, the total test duration is 2.83 DLTs. The application of LID coupled with LLD hybrid approach not only reduces the LEF requirements but also reduces the total test duration. If the impact damage is repaired, the remainder of the test must use the LEFs from the original AS4-PW curve. The load-life shift calculation must now consider the percentage of unsubstantiated design life prior to the repair for calculating the remainder of the test duration.



Figure 3. Load-life-damage (LLD) hybrid approach.

The impact of the reduction in the life shape parameter on the life factor is clearly demonstrated in Figure 3. Thus, the test duration and/or LEF required to demonstrate a certain level of reliability on the DLT or the remaining test life is significantly reduced. However, the risk of structural failure due to large impact can be significantly increased with the extent of the damage. This is addressed in the next section in terms of probability of failure and inspection intervals. Once the new LEF corresponding to the large damage is applied, the spectrum loads of the required test duration can be analyzed in terms of the probability of failure to ensure that the structure can tolerate them, i.e., no or stable damage growth. Inspection intervals can be allotted to monitor the damage state during test to avoid unintentional failure during the test, as large damage has a high probability of growth. In the event that a repair of the impacted damage is deemed necessary to prevent premature failure, then the LEF requirements must be adjusted to reflect the fact that the structure is restored back to its undamaged state.

One possible application to the LLD hybrid approach is illustrated in Figure 4 [7]. This example requires defining ADL and CDT, as well as the necessary inspection interval for damage-tolerant composite structures. Although current certification requirements do not include substantiation of large damages like category 3 and beyond, this approach will help determine load-life enhancement factors related to such a test article with large damage(s). The extra information obtained from such an exercise is beneficial for determining the inspection levels to mitigate risks to the structural integrity as a result of a rare damage threat from a high-energy impact. This approach can be extended also for hybrid structures, as the LEF requirement will be considerably less than the current practice for a composite test article with damage, i.e., LEF of 1.15 for a test duration of 1.5 DLT.



Fatigue (Test Duration) Requirement

Figure 4. Application of LLD hybrid approach for full-scale demonstration.

3.1 Damage Threat Assessment Based on Reliability

In order to ensure no unintentional failure of the structure when using the LLD approach, a reliability-based approach is proposed in this section to evaluate the enhanced spectrum for the remaining test duration after impact resulting in large damage. This approach, based on the fundamental reliability concepts used for both the life-factor and load-enhancement-factor approaches, can be used to evaluate the reliability of damaged test articles and determine the necessary inspection intervals so that the damage is detected prior to it threatening the structural integrity.

Assuming that the residual strength or fatigue life of a composite structure, denoted by the random variable x, follows a two-parameter Weibull distribution, the cumulative distribution function of residual strength or fatigue life can be expressed as

$$P(x;\alpha,\hat{A}) = 1 - \exp\left[-\left(\frac{x}{\hat{A}}\right)^{\alpha}\right]$$
(2)

where \hat{A} is the characteristic residual strength/fatigue life, and α is the shape parameter that determines the scatter of the distribution of random variable x. The shape parameter that corresponds to residual strength or fatigue life is calculated in references [4,5] for several composite material systems used in current aircraft applications. These shape parameters estimate the distribution of strength or life of the full-scale structures. Therefore, the test matrices for determining these parameters must include critical design details and loading parameters that are representative of the full-scale structure. \hat{A} , which is also known as the scale parameter or the location parameter, is calculated as

$$\hat{A} = \left[\frac{1}{n_f} \cdot \sum_{i=1}^{n_f} \left(A_i^{\alpha}\right)\right]^{\frac{1}{\alpha}}$$
(3)

where n_f is the number of data points in the data group. Assuming that the distribution of \hat{A} follows a Chi-squared distribution with 2n degrees of freedom, and α is known, the lower-bound estimate of \hat{A} with a γ -level of confidence is given by [8]

$$\breve{A}_{\gamma} = \hat{A} \cdot \left[\frac{2 \cdot n}{\chi_{\gamma}^{2} (2 \cdot n)} \right]^{1/\alpha}$$
(4)

where the probability of the lower bound estimate is shown as

$$P(\breve{A}_{\gamma} \le \hat{A}) = \gamma \tag{5}$$

The probability of failure (POF) with a γ -level of confidence for an applied stress or fatigue life (\check{A}_R) is shown in equation (6).

$$P(\breve{A}_{R}) = 1 - \exp\left[-\left(\frac{\breve{A}_{R}}{\breve{A}_{\gamma}}\right)^{\alpha}\right]$$
(6)

Given that \check{A}_R is the designed stress or fatigue life of a structure, the reliability of the design (=1-[POF]) with a γ -level of confidence is given in equation (7).

$$R = \exp\left[-\left(\frac{\breve{A}_R}{\breve{A}_{\gamma}}\right)^{\alpha}\right]$$
(7)

For γ =0.95, A- and B-basis reliabilities are 0.99 and 0.90, respectively. Substituting equation (4) for the lower-bound characteristic value in equation (7) and solving for the designed stress or life or the allowable statistics, \check{A}_R , for the desired reliability, R, can be expressed as

$$\breve{A}_{R} = \hat{A} \cdot \left[-\ln(R) \frac{2 \cdot n}{\chi_{\gamma}^{2} (2 \cdot n)} \right]^{1/\alpha}$$
(8)

For a Weibull distribution with an α -shape parameter, the mean value of the population, \overline{A} , is given in equation (9) with respect to the characteristic value.

$$\overline{A} = \hat{A} \cdot \Gamma\left(\frac{\alpha + 1}{\alpha}\right) \tag{9}$$

The scatter factor, the ratio of mean to design (allowable) value, for desired reliability, R, with γ -level of confidence can be expressed as

$$X = \frac{\overline{A}}{\overline{A}_{R}} = \frac{\Gamma\left(\frac{\alpha+1}{\alpha}\right)}{\left[-\ln(R)\frac{2\cdot n}{\chi_{\gamma}^{2}(2\cdot n)}\right]^{1/\alpha}}$$
(10)

The scatter factor signifies the relation between the central tendency of a data set (mean) and the extreme statistics (allowables) as the life factor given in equation (8). The scatter factor for fatigue life and static strength data are referred to as life factor and static factor (S_F), respectively. Solving for the reliability, equation (10) yields

$$R = \exp\left\{-\frac{\chi_{\gamma}^{2}(2 \cdot n)}{2 \cdot n} \cdot \left[\frac{\Gamma\left(\frac{\alpha+1}{\alpha}\right)}{X}\right]^{\alpha}\right\}$$
(11)

and the probability of failure is defined as

$$P_f = 1 - R \tag{12}$$

Equation (10) shows that the reliability of a particular scatter factor depends upon the shape parameter, α , of the data set and the degrees of freedom, 2n, where n is the sample size or, in this case, the number of scaled test articles. Figure 5 shows that the B-basis reliability for DLT is achieved with scatter (life) factors of 13.6 and 4.7 for MSFPs of 1.25 and 2.00, respectively. Similarly, the B-basis reliability on DLL is achieved with scatter (static) factors of 1.15 and 1.10 for modal strength shape parameter (MSSP) of 20 and 30, respectively, indicating that the typical scatter factor of 1.5 on DUL (1.5 · DLL) is more than sufficient to demonstrate B-basis reliability for both of these scatter factors. However, equation (11) does not account for the unintentional deviations from service load, service environmental effects, and structural response variability. The effects of these parameters must be evaluated to completely understand the level of safety provided by the static factor of 1.5.

3.1.1 Cumulative Fatigue Unreliability (CFU) Model

In the event of impact damage to a structure that is designed with a static factor of 1.5, the residual strength is reduced based on the severity of the damage [6]. The reduction to residual strength is denoted by the static-strength reduction factor, δ , and the scatter factor is written as

$$S_F = \frac{\delta \cdot DUL}{DLL} = \delta \cdot \hat{X} \tag{13}$$

where \hat{X} (=1.5) is the static factor prior to the damage. The probability of failure at a fatigue load segment can be determined by combining equations (11), (12), and (13) as

$$P_{f_i} = 1 - \exp\left\{-\frac{\chi_{\gamma}^2(2 \cdot n)}{2 \cdot n} \cdot \left[\frac{\Gamma\left(\frac{\alpha+1}{\alpha}\right)}{\hat{X}_i}\right]^{\alpha}\right\}$$
(14)

where \hat{X}_i is the static factor for i^{th} segment, i.e., ratio of the residual strength and maximum load at i^{th} segment. Also, the initial static factor for a structure is given in equation (15) with the static-strength reduction factor, δ .

$$\hat{X}_0 = \delta \cdot \hat{X} \tag{15}$$

The probability of failure of the structure during a particular fatigue load segment in the spectrum (load sequence) can now be calculated by summing the probability of failure at each segment up to the current segment (n_s) , including the current load segment, as shown in equation (16).



(b) Typical MSSPs

Figure 5. Effects of scatter factor on reliability.

(16)

Since reliability is calculated based on the residual strength degradation or wearout, the sequencing effects are reflected in the cumulative fatigue reliability (CFU) model. When the cumulative probability of failure, P_f , during a load segment in the fatigue spectrum reaches unity, it constitutes the structural failure during that load segment as

$$P_f \ge 1 - TR \to Failure \tag{17}$$

where TR is the target reliability. The CFU model is a measure of the state of a structure with certain damage and certain number of fatigue cycles, but it is not directly related to the damage propagation. Information pertaining to the damage propagation and the residual strength degradation are incorporated into the model through the coupon and element tests, as described in reference [4] for DaDT testing of Starship forward wings.

3.1.2 Considerations for Application of CFU Model

When applying the CFU model to a structural application, several factors need to be considered to accurately predict safe and economical inspection intervals and fatigue life. Because of the robustness of the CFU model, depending on the criticality, i.e., primary load path or redundant structure, and probability of certain damage threat scenarios related to a structure, it can be customized to reduce the amount of test data and computations required to achieve a safe, reliable, and economical DaDT test validation program and inspection intervals.

3.1.2.1 Static-Strength Shape Parameter and Static Factor

For a category 3 damage, the residual strength of the structure will be reduced to its limit load [6], thus δ =2/3, and S_F =1.0. Substituting a static factor for category 3 damage in equation (11), the reliability of the damaged structure can be determined. Consequently, the probability of failure at DLL is calculated using equation (12) and plotted in Figure 6 with respect to MSSP. As can be seen in this figure, the probability of failure is significantly increased for MSSP less than 15. The value of MSSP obtained using the NAVY approach is 20, while it is 32.193 for AS4-PW material based on the scatter analysis in references [4,5]. These values results in 17.4 and 17.8 percent reliability or 82.6 and 82.2 percent probability of failure for DLL, respectively.

It was shown in reference [4] that the static-strength scatter is reduced significantly for damagedelement testing due to stress concentration. Thus, the reliability of a structure with category 3 damage, a population representing less scatter (assume α =30), is compared with the traditionally used MSSP (NAVY approach), α_R =20, and with no impact damage in Figure 7. Although the Bbasis (90 percent with 95 percent confidence) reliability is diminished at DLL, the post-impact reliability of the structure for some operational loads (simulated by spectrum during DaDT test) still remains above the B-basis reliability level, as shown in Figure 7, i.e., the B-basis reliability of a category 3 damaged article (based on α =30) is still maintained for operational loads below 91 percent of DLL, assuming no residual strength degradation.



Figure 6. Probability of failure for S_F =1.0.

Figure 8 compares the post-impact reliability of a category 3 damaged article that belongs to a relatively skewed population (α =30) to the reliability of the undamaged structure that belongs to a population with MSSP of 20 for different operational loads. The B-basis reliability that was maintained for a one-time load application of 130 percent DLL is reduced to 91 percent as a result of the category 3 damage. Since a category 3 damage is expected to be detected within a few flights, it can be repaired and the residual strength of the structure can be restored to DUL.

Note that these reliability calculations do not account for the stiffness degradation or wearout of structural capacity due to repeated loading and do not compare with the one-time application of the operational or applied loads to DLL. It is imperative that residual strength degradation throughout the spectrum is investigated to assess fatigue reliability and consequently the probability of failure.

3.1.2.2 Residual Strength Degradation–Wearout Models

In order to obtain the correct reliability, the residual strength of the structure must be reevaluated after each cycle using a residual strength degradation or wearout technique, and the static factor for the *i*th segment, \hat{X}_i , must be recalculated based on the new residual strength. For a typical aircraft spectrum, this may result in a significant number of calculations, depending on the selection of the wearout model. A closer examination of the reliability results for static strength shape parameters of 20 and 30, shown in Figure 7, reveals that for maximum operational loads below 70 and 80 percent of limit load, respectively, the probability of failure is negligible.



Figure 7. Static-strength reliability at operating loads for a structure with a category 3 damage.



Figure 8. Static-strength reliability comparison before and after impact.

For load cases that are above the truncation levels, a wearout model is required for evaluating the residual strength of the structure after each fatigue load cycle. The Sendeckyj residual strength degradation for constant amplitude fatigue testing can be expressed as a monotonically decreasing function of the number of fatigue cycles, n_f , as shown in equation (18) [Error! Reference source not found.].

$$\sigma_r = \sigma_a \left[\left(\frac{\sigma_e}{\sigma_a} \right)^{\frac{1}{S}} - C(n_f - 1) \right]^{S}$$
(18)

Figure 9 [4] shows a comparison of the residual strength degradation of LID fatigue specimens based on the Sendeckyj wearout model and linear loss of residual strength (LLRS) for n_f constant amplitude fatigue cycles as

$$\sigma_r = \sigma_e + \left(\frac{\sigma_a - \sigma_e}{N_f(\sigma_a)}\right) \cdot n_f \tag{19}$$

where $N_f(\sigma_a)$ is the number of cycles to failure for constant amplitude fatigue loading at the maximum applied cyclic stress, σ_a .



Figure 9. Residual strength degradation for constant amplitude fatigue loading.

When fatigue failure occurs at the n_f^{th} cycle, residual strength is reduced to the maximum applied cyclic stress, and n_f becomes $N_f(\sigma_a)$. Thus, $N_f(\sigma_a)$ can be solved by rearranging the terms in equation (18) as

$$N_f(\sigma_a) = \frac{1}{C} \left[\left(\frac{\sigma_e}{\sigma_a} \right)^{\frac{1}{S}} + C - 1 \right]$$
(20)

As can be seen in Figure 9, when there are not sufficient S-N data to obtain Sendeckyj fitting parameters, the LLRS can be used to conservatively approximate the residual strength, and $N_f(\sigma_a)$ can be obtained using a graphical method from the S-N curve. Note that for both wearout equations (18) and (19), fatigue failure occurs when the residual strength reaches the maximum amplitude fatigue stress level. Further, the CFU model is not restricted to the above two wearout models, but welcomes any appropriate model for calculation of the residual strength after each load cycle. Since these models require a significant number of calculations and most of the loads in a typical fatigue spectrum are below 80 percent of the limit load, a simplified approach is proposed in reference [4]. Once the residual strength is determined, the static factor for the *i*th cycle can be written as

$$\hat{X}_i = \frac{\sigma_{r_i}}{\sigma_{a_i}}$$
(21)

For example, by substituting equation (18) or (19), the scatter factor can be determined. Then it can be substituted into equation (11) to calculate the reliability. Finally, the probability of failure after the corresponding number of fatigue cycles can be calculated for the applied stress. This exercise was carried out for LID S-N data in reference [4], and the scatter analysis of LID S-N data is summarized in Table 1 and illustrated in Figure 10.

The Sendeckyj model was used to fit the S-N data. Residual strength as a function of the number of fatigue cycles was calculated using both the Sendeckyj model and the LLRS. Using the Sendeckyj residual strength, the probability of failure was calculated and is shown in Figure 11. Two stress levels were selected for this simulation: 77.5 and 61 percent of a static failure load that corresponds to 10,000 and 800,000 cycles, respectively. According to the CFU model, the number of cycles corresponding to these two stress levels at 90 percent reliability or 10 percent probability of failure was 9,625 and 799,625, respectively.

4. CONCLUSIONS

The LLD approach introduces the use of multiple LEFs for a particular composite structure, based on the damage category, i.e., use of different LEF curves representing damage severity. The load-life shift calculates the remaining percentage of the design life to be substantiated after completing a certain number of repeated lives with respect to the required repeated lives for the corresponding LEF. Once the test article is inflicted with damage, the remaining test duration is calculated by multiplying the required repeated lives corresponding to the new LEF and the above-mentioned percentage of design life. The example discussed in this paper showed that

this approach not only reduced the LEF requirements for a test article with a large damage but also reduced the remaining test duration as a result of the reduction in data scatter of notched (damaged) composite element test data.

In order to prevent unintentional failure of a damaged article during DaDT testing, especially when investigating extremely improbable high-energy impact threats that reduce the residual strength of a composite structure to its limit load, rigorous inspection intervals are required. The probability of failure of the damaged structure for the enhanced spectrum loads can be evaluated using the cumulative fatigue unreliability (CFU) model proposed in this research. The information obtained from this model can also be used to allot economical and reliable inspection intervals to detect the extent of damage prior to imminent failure or unstable propagation that will threaten the structural integrity. This approach can also be extended to determine the inspection interval during service based on a target reliability and a critical damage threshold.

	Static Strength		Durability and Damage Tolerance			
Damage Category	Weibull Analysis		Sendeckyj Analysis		Weibull Analysis	
	α	β	With Static	Without Static	Individual	Joint
BVID	45.771	36413	1.774	2.234	2.446	2.355
VID	32.222	30103	2.182	2.658	2.991	2.779
LID	36.676	25776	2.466	2.799	3.272	3.250

Table 1. Scatter analysis results for damage elements



Figure 10. Comparison of DTE life shape parameters.



Figure 11. Example of CFU model for constant amplitude fatigue tests.

Realistic target reliability must be used for determination of inspection intervals accounting the safety and the cost considerations. Although category 3 damage is recommended for determining the inspection intervals, more realistic damage threat levels can be used considering the probability of occurrence so that more practical or economical inspection levels can be determined. In order to further extend the CFU model for determination of inspection intervals for a fleet, the fleet size and the probability of detectability may have to be considered in addition to the above-mentioned parameters.

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